JOULE LOSS DUE TO COMPRESSIBILITY OF GAS IN A CHANNEL OF VARIABLE SECTION

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It is known that closed electric currents arise in a conducting medium moving in a nonuniform magnetic field. These currents lead to additional energy loss and adversely affect the characteristics of magnetohydrodynamic channels. (The numerous investigations of these effects are dealt with in the review [2, 3].) Eddy electric currents are also formed, however, when a medium flows in a uniform magnetic field perpendicular to the to the plane of motion if the channel has a variable cross section and the medium is compressible [1]. This paper is devoted to an investigation of some features of these flows. It is assumed in the analysis that the gas flows in channels whose geometry varies slightly.

1. We consider the movement of a conducting, nonviscous, and nonheat-conducting gas in a plane channel $|x^0| < \infty$, $h_1(x^0) < y < h_* + h_2(x^0)$ ($h_* = \text{const} > 0$) in the presence of a uniform transverse magnetic field $B = 0, 0, B_*$). We assume that the induced magnetic field can be neglected, the electrical conductivity σ and the Hall parameter β for electrons are constant, and the sliding of ions is not significant. We assume that the top and bottom walls of the channel deviate slightly from the surfaces $y = h_*$ and $y = 0$, respectively, and these deviations vary slowly along the flow. This condition is written mathematically in the following way:

$$\frac{h_1(x^\circ)}{h_*} = \varepsilon f^-(x), \quad \frac{h_2(x^\circ)}{h_*} = \varepsilon f^+(x) \quad \left(x = \frac{x^\circ}{h_*}\right). \tag{1.1}$$

Here ε = const is a small parameter characterizing the deviation of the geometry from a channel of constant cross section and the functions f^-, f^+, and their derivatives are of order of unity. We will assume that functions f^- and f^+ are zero when $x = -\infty$ (and are bounded when $x = \infty$).

Henceforth we will consider flows characterized by a uniform distribution of gasdynamic parameters when $x = -\infty$. If the channel cross section does not change with x^0 (the flow occurs in a channel $0 < y^0 < h_*$), these uniform distributions of parameters will be preserved in any cross section. In this there will be separation of the electric charge in the channel, and the density $\mathbf{j}$ of the electric current will be zero.

When the channel cross section varies with x^0, electric currents arise in the moving gas (the parameters of which are nonuniform) and electromagnetic forces begin to act on the gas. If the channel geometry satisfies conditions (1.1) the deviations of the gasdynamic parameters from the uniform distributions at $x = -\infty$ and the electric currents (and forces) arising are of the order of ε. Thus, magnetogasdynamic equations can be linearized with respect to ε near the solution for a channel of constant section, which, as was mentioned above, has the form

$$u = 1, \quad v = 0, \quad \rho = 1, \quad p = p_* = \text{const}, \ \varphi = -y, \quad j_x = 0, j_y = 0. \tag{1.2}$$

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Here and henceforth the values of the axial and transverse velocities u and v relate to the characteristic velocity U, the components j_x and j_y to $\sigma U B_*/c$ (c is the speed of light in vacuum), the electric potential to UB_*h_*/c, the density ρ and pressure p to ρ_* and $\rho_* U^2$(ρ_* is the characteristic density), and the coordinates x and y to the height h_*.

The solution of the problem can be sought in the form of the following series in powers of ε:

$$\begin{aligned} u &= 1 + \varepsilon u_1(x, y) + \ldots, & v &= \varepsilon v_1(x, y) + \ldots \\ \rho &= 1 + \varepsilon \rho_1(x, y) + \ldots, & p &= p_* + \varepsilon p_1(x, y) + \ldots \\ \varphi &= -y + \varepsilon \varphi_1(x, y) + \ldots, & \mathbf{j} &= \varepsilon \mathbf{j}_1(x, y) + \ldots \end{aligned} \tag{1.3}$$

Substituting (1.3) in the system of magnetogasdynamic equations and using assumptions adopted for the first approximation, we find the following system of equations:

$$\frac{\partial u_1}{\partial x} + \frac{\partial p_1}{\partial x} = s j_{y1}, \qquad \frac{\partial v_1}{\partial x} + \frac{\partial p_1}{\partial y} = -s j_{x1},$$
$$\frac{\partial \rho_1}{\partial x} + \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \qquad \frac{\partial p_1}{\partial x} - \frac{1}{M^2}\frac{\partial \rho_1}{\partial x} = 0,$$
$$\left(s = \frac{\sigma B_*^2 h_*}{c^2 \rho_* U}, \ M = (\gamma p_*)^{-1/2}\right) \tag{1.4}$$

$$j_{x1} = \frac{1}{1+\beta^2}\left[-\frac{\partial \varphi_1}{\partial x} + v_1 + \beta\left(u_1 + \frac{\partial \varphi_1}{\partial y}\right)\right],$$
$$j_{y1} = \frac{1}{1+\beta^2}\left[-\frac{\partial \varphi_1}{\partial y} - u_1 - \beta\left(\frac{\partial \varphi_1}{\partial x} - v_1\right)\right],$$
$$\frac{\partial j_{x1}}{\partial x} + \frac{\partial j_{y1}}{\partial y} = 0. \tag{1.5}$$

In (1.4) the first two relationships will be the projections of the momentum equation on the x- and y-axes, the third expression will be the continuity equation, and the fourth the energy equation, according to which the entropy of the gas is constant within a first approximation. It follows that the Joule dissipation is a quantity of the order of ε^2.

The dimensionless quantities s and M are the parameters of magnetogasdynamic interaction and Mach number.

The relationships in (1.5) are derived from Ohm's law and the continuity equation for the electric current.

It follows from the first two equations of (1.3) and the last relationship in (1.5) that

$$\frac{\partial \omega}{\partial x} = 0 \qquad \left(\omega = \frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y}\right). \tag{1.6}$$

Here ω is the vorticity.

Since $\omega = 0$ when $x = -\infty$, then, according to (1.6), the flow in the channel will be vortex-free and we can introduce a velocity potential

$$u_1 = \frac{\partial \Phi}{\partial x}, \qquad v = -\frac{\partial \Phi}{\partial y}. \tag{1.7}$$

After simple transformations we obtain equations for functions Φ and φ from (1.4)–(1.5):

$$(1 - M^2)\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = \frac{sM^2}{1+\beta^2}\left[\frac{\partial \varphi_1}{\partial y} + \frac{\partial \Phi}{\partial x} + \beta\left(\frac{\partial \varphi_1}{\partial x} - \frac{\partial \Phi}{\partial y}\right)\right], \tag{1.8}$$

$$\Delta \varphi_1 = \beta \Delta \Phi. \tag{1.9}$$

We assume that the channel walls are ideal insulators and impermeable to the gas. After linearization the boundary conditions on the walls are written in the following way:

$$\frac{\partial \varphi_1}{\partial y} + \frac{\partial \Phi}{\partial x} + \beta\left(\frac{\partial \varphi_1}{\partial x} - \frac{\partial \Phi}{\partial y}\right) = 0 \quad \text{when } y = 0 \text{ and } y = 1 \tag{1.10}$$

$$\frac{\partial \Phi}{\partial y} = f^{+1}(x) \quad \text{when } y = 1, \tag{1.11}$$

$$\frac{\partial \Phi}{\partial y} = f^{-1}(x) \quad \text{when } y = 0. \tag{1.12}$$

Equation (1.8) is of the elliptic type when $M < 1$ and of the hyperbolic type when $M > 1$. When $M \approx 1$ the linear theory, as is known [4], is inapplicable.

In the case of an incompressible liquid ($M=0$) the function Φ is harmonic and is found independently [by using the boundary conditions (1.11) and (1.12)] of the relationships containing the electromagnetic terms. The electric potential φ and Φ will be the real and imaginary parts of the analytical function [here the boundary condition (1.10) is satisfied], i.e.,

$$\frac{\partial \varphi_1}{\partial x} = \frac{\partial \Phi}{\partial y}, \qquad \frac{\partial \varphi_1}{\partial y} = -\frac{\partial \Phi}{\partial x}. \tag{1.13}$$

Substituting (1.13) in system (1.5) we find that $j_{x_1} \equiv 0$, $j_{y_1} \equiv 0$, i.e., when $M=0$ the change in the shape of the cross section does not give rise to electric currents. This is due to the fact that when B=const, the induced potential difference in a channel with nonconducting walls depends only on the volume flow rate of gas, which does not vary along the channel when $M=0$. Hence, the derived conclusion will still be valid for any (not only a slow) change in channel shape and any (not necessarily vortex-free) flow at the outlet.

To prove this assertion we note that in an incompressible fluid components j_x and j_y can be written in the form [5, 2]

$$j_x = \frac{1}{1+\beta^2}\left(-\frac{\partial \Omega}{\partial x} + \beta\frac{\partial \Omega}{\partial y}\right), \quad j_y = \frac{1}{1+\beta^2}\left(-\frac{\partial \Omega}{\partial y} - \beta\frac{\partial \Omega}{\partial x}\right), \quad \Delta\Omega = 0, \tag{1.14}$$
$$(\Omega = \varphi + \psi).$$

Hence ψ is the fluid current function. On the channel walls (which are assumed to be nonconducting) the condition $\partial\Omega/\partial n = -\beta\partial\Omega/\partial\tau$ is fulfilled (τ and n are the unit vectors of the tangent and normal to the wall). The formulated curve problem has solution* Ω=const; hence, it follows that $j \equiv 0$.

If the medium is incompressible, the volume flow rate G will not remain constant if the shape of the section changes. When $M < 1$ the value of G decreases or increases, depending on whether the cross-sectional area F increases or decreases along the channel†. If the flow is supersonic, then $dG/dx > 0$ when $dF/dx > 0$ and $dG/dx < 0$ when $dF/dx < 0$. Hence, the induced potential difference between the bottom and top channel walls varies with x, and this gives rise to eddy electric currents.

The system of Eq. (1.8)–(1.12), from which gasdynamic and electromagnetic parameters are determined in the first approximation, can be solved with the aid of the Fourier transform.

2. We consider a flow with weak magnetohydrodynamic interaction when $s \ll 1$. In this case the potential distribution is found from the well-known equation of linear gas dynamics,

$$(1 - M^2)\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0. \tag{2.1}$$

We assume that the bottom wall of the channel is defined by the equation $y=0$, i.e., $f^- \equiv 0$.

We first investigate subsonic flows ($\delta^2 = 1 - M^2 > 0$).

*We note that if part of the channel walls is formed by the electrodes the solution Ω=const in the general case does not satisfy the physical content of the problem. The solution for j_x and j_y must be derived from a class of functions not bounded close to the contact points of the electrodes and insulators and in which the form of the boundary conditions changes.

†The indicated change in G occurs in the case of a weak magnetogasdynamic interaction.

According to the asymptotic condition at x=−∞ and the assumption of a bound for the function $f^+(x)$ at x= ∞ we have

$$|\Phi(x, y)| < \text{const}\exp(\tau_+ x), \quad |\varphi_1(x, y)| < \text{const}\exp(\tau_+ x) \quad \text{when} \quad x \to -\infty;$$
$$|\Phi(x, y)| < \text{const}\exp(\tau_- x), \quad |d_1(x, y)| < \text{const}\exp(\tau_- x) \quad \text{when} \quad x \to \infty;$$
$$0 < \tau_- < \tau_+ . \tag{2.2}$$

Equations (2.2) reflect the fact that functions Φ and φ tend exponentially to zero when $x \to -\infty$ and increase much more slowly when $x \to \infty$. More precisely, when $x \to \infty$ φ is always bounded and the potential Φ varies linearly with x if $f^+(x) \to \text{const} \neq 0$ when $x \to \infty$ and bounded if the function $f^+(x)$ at large x oscillates about some constant value.

For the solution of system (2.1) and (1.9)-(1.12) we use the Fourier transform

$$\xi^\circ(y, \alpha) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \xi(x, y)\exp(i\alpha x)\,dx \quad (a = \nu + i\tau,\ \tau_- < \tau < \tau_+). \tag{2.3}$$

By the function ξ here we mean the functions $\Phi(x,y)$, $\varphi_1(x,y)$, and $f^+(x)$. According to (2.2), ξ^0 will be an analytical function of argument α in the band $\tau_- < \tau < \tau_+$ [6].

The system of equations for the images of the required functions has the form

$$-\delta^2\alpha^2\Phi^\circ + \frac{d^2\Phi^\circ}{dy^2} = 0, \quad -\alpha^2\varphi_1{}^\circ + \frac{d^2\varphi_1{}^\circ}{dy^2} = \beta\left(-\alpha^2\Phi^\circ + \frac{d^2\Phi^\circ}{dy^2}\right),$$
$$\Phi^{\circ\prime}(0, \alpha) = 0, \quad \Phi^{\circ\prime}(1, \alpha) = t(\alpha) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} (df^+/dx)\exp(i\varkappa x)\,dx,$$
$$\varphi_1{}^{\circ\prime} - i\alpha\Phi^\circ - \beta(i\alpha\varphi_1{}^\circ + \Phi^{\circ\prime}) = 0 \quad \text{for} \quad y = 0 \text{ and } y = 0. \tag{2.4}$$

Here the prime denotes the derivative with respect to y.

The solution of system (2.4) is given by

$$\Phi^\circ = \frac{t(\alpha)\,\text{ch}(\alpha\delta y)}{\alpha\delta\,\text{sh}(\alpha\delta)}, \quad \varphi_1{}^\circ = \frac{it(\alpha)}{\alpha\delta\,\text{sh}(\alpha\delta)\,\text{sh}\,\alpha}\{\text{ch}(\alpha\delta)\,\text{ch}(\alpha y) - \text{ch}[\alpha(1-y)] +$$
$$+ i\beta[\text{sh}[\alpha(1-y)] + \text{ch}(\alpha\delta)\,\text{sh}(\alpha y) - \text{ch}(\alpha\delta y)\,\text{sh}\,\alpha]\}. \tag{2.5}$$

The originals $\Phi(x, y)$ and $\varphi_1(x,y)$ are found by means of the inversion theorem [6]

$$\Phi = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty+i\tau^*}^{\infty+i\tau^*} \Phi^\circ\exp(-i\alpha x)\,d\alpha, \quad \varphi_1 = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty=i\tau^*}^{\infty+i\tau^*} \varphi_1{}^\circ\exp(-i\alpha x)\,d\alpha$$
$$(0 < \tau_- < \tau^* < \tau_t). \tag{2.6}$$

Integral (2.6) after specification of the function $t(\alpha)$ can be calculated by means of the residue theorem.

Let the function $f^+(x)$ have the following form (Fig. 1):

$$f^+(x) = \begin{cases} 0 & \text{when } x < 0 \\ \sin kx & \text{when } x > 0 \end{cases} \tag{2.7}$$

Here $t(\alpha)$ is given by

$$t(\alpha) = -\frac{i\alpha k}{\sqrt{2\pi}(k^2 - \alpha^2)} \tag{2.8}$$

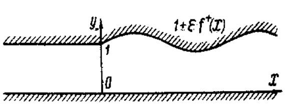

Fig. 1

Substituting (2.8) into Eqs. (2.6) and using the residue theorem and Jordan's lemma [7] we find that for calculation of the integrals (2.6) when $x > 0$ we have to sum the residues of the integrands in the region $\tau < \tau^*$, and when $x < 0$ we have to sum the residues in the region $\tau > \tau^*$.

As a result of the calculations we find:

$$\Phi_{x>0} = \Phi^*(x, y) + \frac{k}{\delta^2} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos(n\pi y)\exp(-n\pi x/\delta)}{k^2 + (n\pi/\delta)^2}$$

$$\varphi_1 = \varphi^*(x, y) + \frac{k}{\delta^2}\Sigma_1 + \frac{k}{\delta}\Sigma_2 \qquad \Phi^*(x, y) = \frac{\cos(kx)\,\mathrm{ch}(k\delta y)}{\delta\,\mathrm{sh}\,k\delta} - \frac{1}{k\delta^2} \tag{2.9}$$

$$\varphi^*(x, y) = \frac{1}{\delta\,\mathrm{sh}\,k\,\mathrm{sh}\,k\delta}\{\sin kx\,[\mathrm{ch}(k\delta)\,\mathrm{ch}(ky)] - \mathrm{ch}[k(1-y)] -$$
$$- \beta\cos kx\,[\mathrm{sh}[k(1-y)] + \mathrm{ch}(k\delta)\,\mathrm{sh}(k\delta y)\,\mathrm{sh}\,k]\} \tag{2.10}$$

$$\Sigma_1 = \sum_{n=1}^{\infty} (-1)^n \frac{\exp(-r_n x)\,[\chi_n(y) + \beta\varkappa_n(y)]}{(k^2 + r_n^2)\sin r_n}, \quad r_n = \frac{n\pi}{\delta}$$

$$\chi_n = (-1)^n \cos(r_n y) - \cos[r_n(1-y)]$$

$$\varkappa_n = \sin[r_n(1-y)] + (-1)^n \sin(r_n y) - \cos(r_n \delta y)\sin r_n \tag{2.11}$$

$$\Sigma_2 = \sum_{n=1}^{\infty} (-1)^n \frac{\exp(-q_n x)\,[\mu_n(y) + \beta\nu_n(y)]}{(k^2 + q_n^2)\sin(q_n\delta)} \quad q_n = n\pi$$

$$\mu_n = \cos(q_n\delta)\cos(q_n y) - \cos[q_n(1-y)]$$

$$\nu_n = \sin[q_n(1-y)] + \cos(q_n\delta)\sin(q_n y)$$

$$\Phi_{x<0} = \frac{k}{\delta^2}\sum_{n=1}^{\infty}(-1)^n \frac{\cos(r_n\delta y)\exp(r_n x)}{k^2 + r_n^2}$$

$$\varphi_{1\,x<0} = \frac{k}{\delta^2}\sum_{n=1}^{\infty} \frac{(-1)^n \exp(r_n x)\,[\chi_n(y) - \beta\varkappa_n(y)]}{(k^2 + r_n^2)\sin r_n} + \frac{\varkappa}{\delta}\sum_{n=1}^{\infty} \frac{(-1)^n \exp(q_n x)\,[\mu_n(y) - \beta\nu_n(y)]}{(k^2 + q_n^2)\sin q_n\delta}. \tag{2.12}$$

The functions $\Phi^*(x, y)$ and $\varphi^*(x, y)$ given by (2.10) are the periodic nondecaying part of the solution at large x and due to sinusoidal variation of the channel profile. The currents j_x^* and j_y^* corresponding to this part of the solution are given by the formulas

$$j_x^* = \frac{k\cos kx}{\delta\,\mathrm{sh}(k\delta)\,\mathrm{sh}\,k}[\delta\,\mathrm{sh}(k\delta y)\,\mathrm{sh}\,k - \mathrm{ch}(k\delta)\,\mathrm{ch}(ky) + \mathrm{ch}[k(1-y)]]$$
$$j_y^* = \frac{k\sin kx}{\delta\,\mathrm{sh}(k\delta)\,\mathrm{sh}\,k}[\mathrm{ch}(k\delta y)\,\mathrm{sh}\,k - \mathrm{ch}(k\delta)\,\mathrm{sh}(ky) - \mathrm{sh}[k(1-y)]]. \tag{2.13}$$

From (2.13), the distribution of currents j_x^* and j_y^* (as distinct from the potential) is independent of the Hall parameter β.

The Joule dissipation q in the volume of the channel $0 < y < 1$, $(N\pi/k) < x < (N+1)\pi/k$ $(N \gg 1)$, corresponding to the wave halflength π/k, is given by the expression

$$q = \frac{\pi k}{2\delta^2\,\mathrm{sh}^2 k\delta}\left[\frac{1+\delta^2}{4k\delta}\,\mathrm{sh}(2k\delta) + \frac{1-\delta^2}{2} + (1 + l^2)\frac{\mathrm{sh}(2k)}{2k} -\right.$$
$$\left. - \frac{4l}{k}\,\mathrm{ch}\,k\,\mathrm{sh}[{}^1/_2 k(1+\delta)]\,\mathrm{sh}[{}^1/_2 k(1-\delta)] - \frac{2}{k}\,\mathrm{sh}\,k\,\mathrm{ch}(k\delta)\right]$$
$$\left(q = \int_0^1 \int_0^{\lambda/\lambda} (j_x^{*2} + j_y^{*2})\,dx\,dy, \quad l = \frac{\mathrm{ch}\,k - \mathrm{ch}(k\delta)}{\mathrm{sh}\,k}\right). \tag{2.14}$$

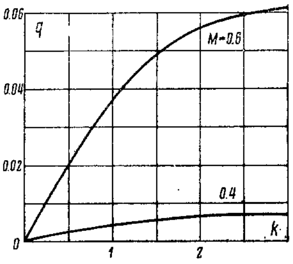

Fig. 2

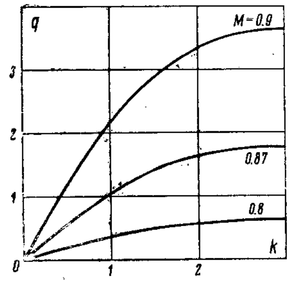

Fig. 3

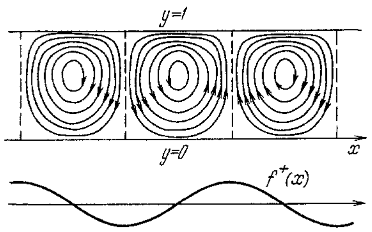

Fig. 4

Relationships q(k) for different Mach numbers are shown in Figs. 2 and 3. With increase in M the Joule loss increases. The greatest increase in the dissipation q occurs when M is close to unity. It should be borne in mind, however, that the linear approximation is less accurate in this case. With reduction in k the Joule dissipation tends to zero, since the rate of change of the channel profile is reduced.

With increase in k the Joule dissipation within a halfwave (and in any fixed volume of the channel) increases. As (2.14) shows, $q \to \infty$ when $k \to \infty$. Linear theory and (2.14) derived from it, however, are inapplicable at large k owing to the unbounded increase in the modulus of the derivative $f^{+\prime}(x)$. The formal explanation of the unbounded increase in q when $k \to \infty$ is that reduction of the wavelength $1/\pi k$ is accompanied by an increase in the kinetic energy of the flow.

The electric current lines in the channel at large x are shown in Fig. 4. The greatest potential difference between the bottom and top walls is induced in the cross sections $x_{1N}=(3/2\pi \times 2N\pi)/k$, and the least in $x_{2N}=(1/2\pi + 2N\pi)/k$ (N is a large whole number). In the cross sections x_{1N} the gas flow is characterized by the greatest, and in cross sections x_{2N} by the least, volume flow rate. Electric current distribution agrees with the mentioned features of the potential distribution.

3. We consider a supersonic gas flow (M > 1). We assume that $f^-(x) \equiv 0$, $s=0$, $\beta=0$, and $f^+(x)$ is given by (2.7). By means of the Fourier transform we find the images Φ^0 and φ_1^0 of the functions and then their originals

$$\Phi = -\frac{k}{2\pi i\delta}\int_{-\infty+i\tau^*}^{\infty+i\tau^*}\frac{\cos(\alpha\omega y)\exp(-i\alpha x)\,d\alpha}{(k^2-\alpha^2)\sin(\alpha\omega)} \quad (\omega^2 = M^2-1,\ 0<\tau_-<\tau^*<\tau_+),$$

$$\varphi_1 = \frac{k}{2\pi i\delta}\int_{-\infty+i\tau^*}^{\infty+i\tau^*}\frac{i\,[\operatorname{ch}[\alpha(1-y)] - \cos(\alpha\omega)\operatorname{ch}(\alpha y)]\exp(-i\alpha x)\,d\alpha}{(k^2-\alpha^2)\operatorname{sh}\alpha\sin(\alpha\omega)}. \qquad (3.1)$$

Using the residue theorem we find

$$\Phi_{x>0} = \frac{1}{k\omega^2} - \frac{\cos(k\omega y)\cos(kx)}{\omega\sin(k\omega)} + \frac{2k}{\omega^2}\sum_{n=1}^{\infty}(-1)^n\frac{\cos(r_n\omega y)\cos r_n x}{k^2-r_n^2} \quad \left(r_n = \frac{n\pi}{\omega}\right). \qquad (3.2)$$

$$\underset{x>0}{\varphi_1} = \frac{\sin(kx)\,[\mathrm{ch}\,[k(1-y)] - \cos(k\omega)\,\mathrm{ch}\,(ky)]}{\omega\,\mathrm{sh}\,k\,\sin(k\omega)}$$

$$+\frac{2k}{\omega^2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(r_n x)\,[\mathrm{ch}\,[r_n(1-y)] + (-1)^{n+1}\mathrm{ch}\,(r_n y)]}{(k^2 - r_n^2)\,\mathrm{sh}\,r_n}$$

$$+\frac{k}{\omega}\sum(-1)^n\frac{\exp(-q_n x)\,\zeta_n(y)}{(k^2+q_n^2)\,\mathrm{sh}\,(q_n\omega)}, \qquad (3.3)$$

$$\underset{x<0}{\Phi} = 0, \qquad \underset{x<0}{\varphi_1} = \frac{k}{\omega}\sum_{n=1}^{\infty}\frac{(-1)^n\exp(q_n x)\,\zeta_n(y)}{(k^2+q_n^2)\,\mathrm{sh}\,(q_n\omega)}$$

$$(q_n = n\pi,\ \zeta_n(y) = \cos[q_n(1-y)] - \mathrm{ch}\,(q_n\omega)\cos(q_n y)). \qquad (3.4)$$

It was assumed in deriving (3.2-(3.4) that k does not coincide with any value of r_n.

As the obtained formulas show, the velocity distribution at large x depends not only on the "local" geometry of the channel [the first two terms in (3.2)], but also on the perturbations which are propagated, according to the characteristics, downstream from the cross section x=0 and are repeatedly reflected from the channel walls (the third term in this formula).

Despite the fact that there are no velocity perturbations at $x<0$ (the velocity perturbations are zero left of the characteristic $y=1-x\omega^{-1}$), the electric currents in this region differ from zero.

According to (3.4), the current j_x is negative for $y=0$ and positive for $y=1$. This is consistent with the fact that in the region $0<x<1/2\pi k^{-1}$ the potential difference between the bottom and top walls increases steadily with increase in x.

The volume flow rate G of the gas on the basis of (1.2) and (3.2) is given by the formula

$$G = \int_0^1 u\,dy = \begin{cases} 1 \text{ when } x<0 \\ 1+\varepsilon\sin(kx)/\omega^2 \text{ when } x>0. \end{cases}$$

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